Exam
Mechanics & Relativity 2017–2018 (part Classical Mechanics)
October 31, 2017

INSTRUCTIONS

- Write the name of your tutor and/or group on the top right-hand corner of the first sheet handed in, and *drop your work in the box with your tutor's name.*
- This exam comprises 4 problems. Start your solution of each problem on a separate sheet.
- The first is a set of five conceptual multiple-choice questions, for which only the answer matters not your arguments. The answers to problems 2 through 4 require clear arguments and derivations, all written in a well-readable manner.
- The number of points for every subquestion is indicated inside a box in the margin. The total number of points per problem is

Problem	# of points
1	5
2	6
3	6
4	4

and the grade is computed as (total # points) $/21 * 9 + 1$.

Some useful formulas

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi\,, \quad \cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi\,, \quad e^{a+b} = e^a e^b\,;$$

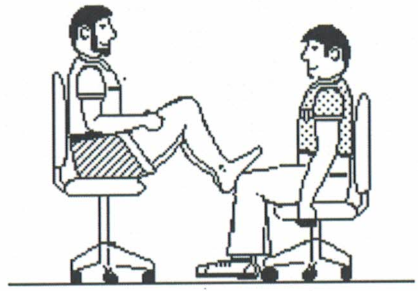
$$\frac{d}{dt}\sin(\alpha t) = \alpha\cos(\alpha t)\,, \qquad \frac{d}{dt}\cos(\alpha t) = -\alpha\sin(\alpha t)\,, \qquad \frac{d}{dt}e^{\alpha t} = \alpha e^{\alpha t}\,.$$

The solution for x from the quadratic equation $ax^2 + bx + c = 0$ is given by

$$x = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

Problem 1 (5 points) Below are five conceptual multiple-choice questions (a thru e). Only the answer (A ... E) matters, not your arguments.

a. In the figure on the right, student "a" has a mass of 95 kg and student "b" has a mass of 77 kg. They sit in identical office chairs facing each other. Student "a" places his bare feet on the knees of student "b", as shown. Student "a" then suddenly pushes outward with his feet, causing both chairs to move. During the push and while the students are still touching one another:



A) neither student exerts a force on the other.

B) student "a" exerts a force on student "b", but "b" does not exert any force on "a".

C) each student exerts a force on the other, but "b" exerts the largest force.

D) each student exerts a force on the other, but "a" exerts the largest force.

E) each student exerts the same amount of force on the other. 1

b. Despite a very strong wind, a tennis player manages to hit a tennis ball with her racquet so that the ball passes over the net and lands in her opponents court.

Consider the following forces:

1. a downward force of gravity.

2. a forward force by the "hit"

3. a force exerted by the air.

Which of the above forces is (are) acting on the tennis ball after it has left contact with the racquet and before it touches the ground?

A) 1 only.

B) 1 and 2.

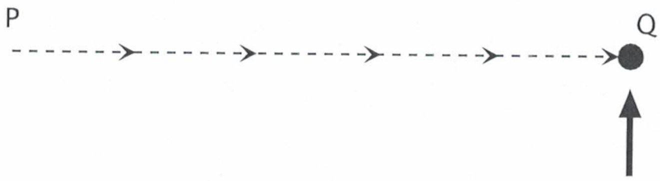
C) 1 and 3.

D) 2 and 3.

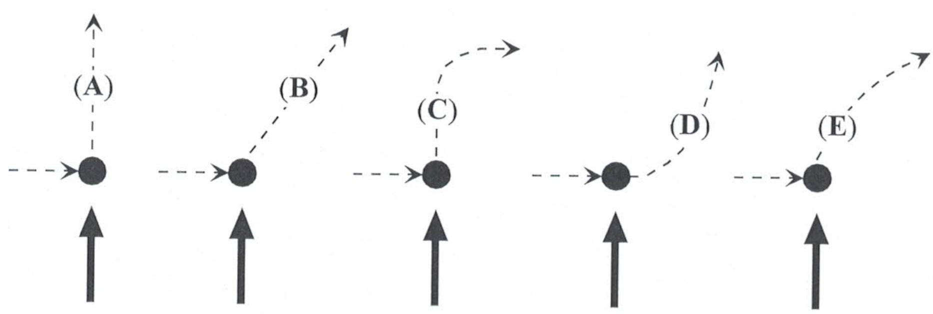
E) 1, 2 and 3. 1

Use the statement and figure below to answer the next *two* multiple-choice questions.

The figure depicts a hockey puck sliding with constant speed v_0 in a straight line from point P to point Q on a frictionless horizontal surface. Forces exerted by the air are negligible. You are looking down on the puck. When the puck reaches point Q, it receives a swift horizontal kick in the direction of the heavy print arrow. Had the puck been at rest at point Q, then the kick would have set the puck in motion with a speed v_k in the direction of the kick.



c. Which of the paths below would the puck most closely follow after receiving the kick?



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d. The speed of the puck just after it receives the kick is:

A) equal to the speed v_0 it had before the kick.

B) equal to the speed v_k resulting from the kick and independent of v_0.

C) equal to the arithmetic sum of the speeds v_k and v_0.

D) smaller than either of the speeds v_k and v_0.

E) greater than either of the speeds v_k and v_0, but less than $v_k + v_0$.

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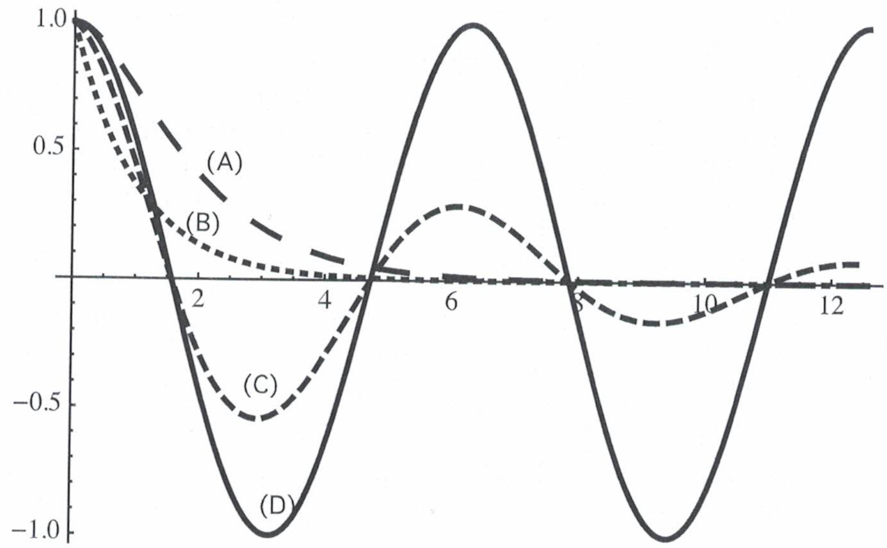
e. Which of the curves A, B, C or D in the figure below characterises a critically damped oscillator?



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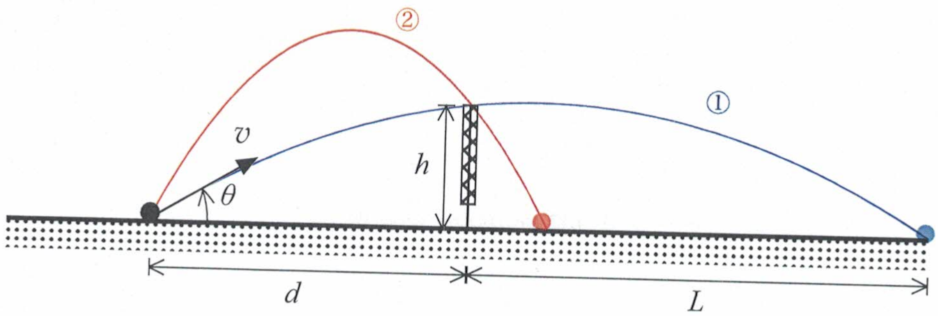
The answers to problems 2 through 4 require clear arguments and derivations, all written in a well-readable manner.

Problem 2 (6 points) An important skill in playing (table) tennis is the ability to let the ball fly just over the net. In doing so, one strategy is to make the ball land all the way at the end of the opponent's quarter; another strategy, instead, is to make the ball drop as close to the net as possible. This problem is concerned with an idealised version, where the dimension of the ball and any air resistance is neglected.



A ball is launched at an angle θ with respect to the ground with initial speed v as indicated in the figure above. The distance to the net is d, its height is h. The figure shows two trajectories for which the ball just touches the top of the net.

a. Forget about the net for this moment. Prove that the time t_0 at which the ball lands on the floor again is given by $t_0 = (2v\sin\theta)/g$. 1

b. Now, consider that the ball is supposed to fly over the net. Suppose that the velocity v is given. How large should θ be such that the ball can reach at least the height h? 2

c. Show that the time t_1 at which the ball is right on top of the net for trajectory ② is given by

$$t_1 = \frac{1}{2}t_0 - \sqrt{(t_0/2)^2 - 2h/g}\,.$$

3

Problem 3 (6 points) A particle of mass m is attached to a linear spring with stiffness k, and is subject to a driving force $F_d \cos \omega t$. But there is no damping!

a. Show that the equation of motion for the particle can be written as

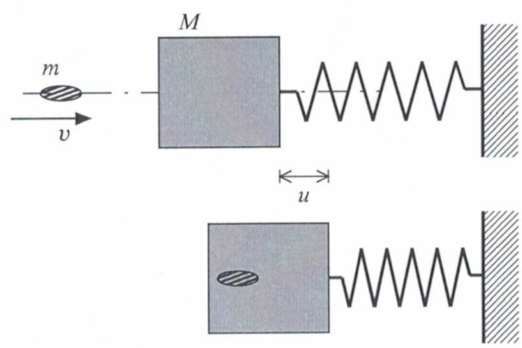
$$\ddot{x} + {\omega_0}^2 x = F \cos \omega t \,,$$

with $\omega_0 = \sqrt{k/m}$ and $F = F_d/m$. [1]

b. This linear differential equation, but with right-hand side = 0, would have a homogeneous solution in the form of the usual superposition of $\cos \omega_0 t$ and $\sin \omega_0 t$. In addition it needs a particular solution to match the right-hand side. In view of the time dependence in the right-hand side, we seek this particular solution by guessing $x(t) = A \cos \omega t + B \sin \omega t$. For what values of A and B is this a possible solution? [2]

c. If you would write this in the form $x(t) = C \cos (\omega t - \phi)$, with $C > 0$, what are C and ϕ? Be careful about the phase, because there are two cases to be considered. [3]

Problem 4 (4 points) In order to measure the velocity of a bullet (mass m), it is fired into a wooden block with mass M that is attached to a wall by means of a spring with stiffness k. The spring will be compressed after the bullet has penetrated the block. The original velocity of the bullet can be determined from the maximum compression of the spring.



a. Determine the velocity of the bullet and block directly after impact. Which conservation law or laws apply here? [1]

b. After impact, block+bullet will move in a harmonic fashion. At which frequency? [1]

c. Express the initial velocity of the bullet in terms of the maximum compression of the spring, u. [2]

Please note corrections in Problem 2 as indicated below

c. Show that the time $\boldsymbol{t_2}$ at which the ball is right on top of the net for trajectory ② is given by

$$t_2 = \frac{1}{2}t_0 + \sqrt{(t_0/2)^2 - 2h/g}\,.$$

3